

# Icosahedral Reconstruction

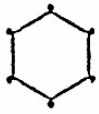
Wen Jiang



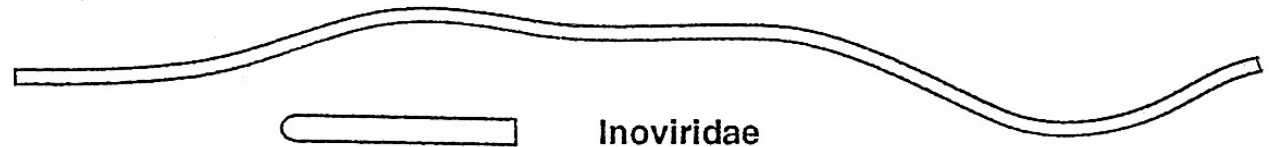
**PURDUE**  
UNIVERSITY™

# Icosahedral Viruses: Bacteriophages

**ssDNA**

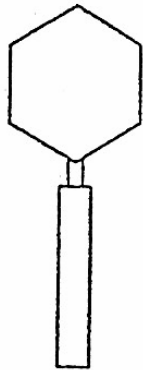


Microviridae

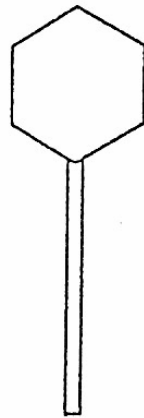


Inoviridae

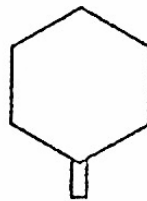
**dsDNA**



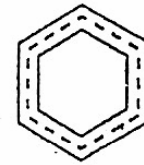
Myoviridae



Siphoviridae



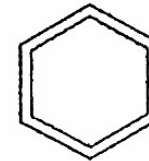
Podoviridae



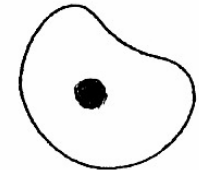
Corticoviridae



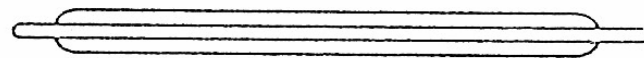
Fuselloviridae



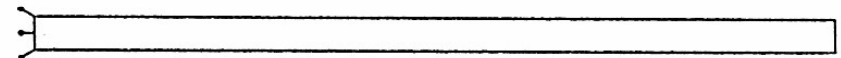
Tectiviridae



Plasmaviridae



Lipothrixviridae



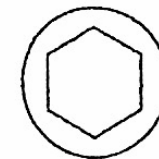
Rudiviridae

**ssRNA**



Leviviridae

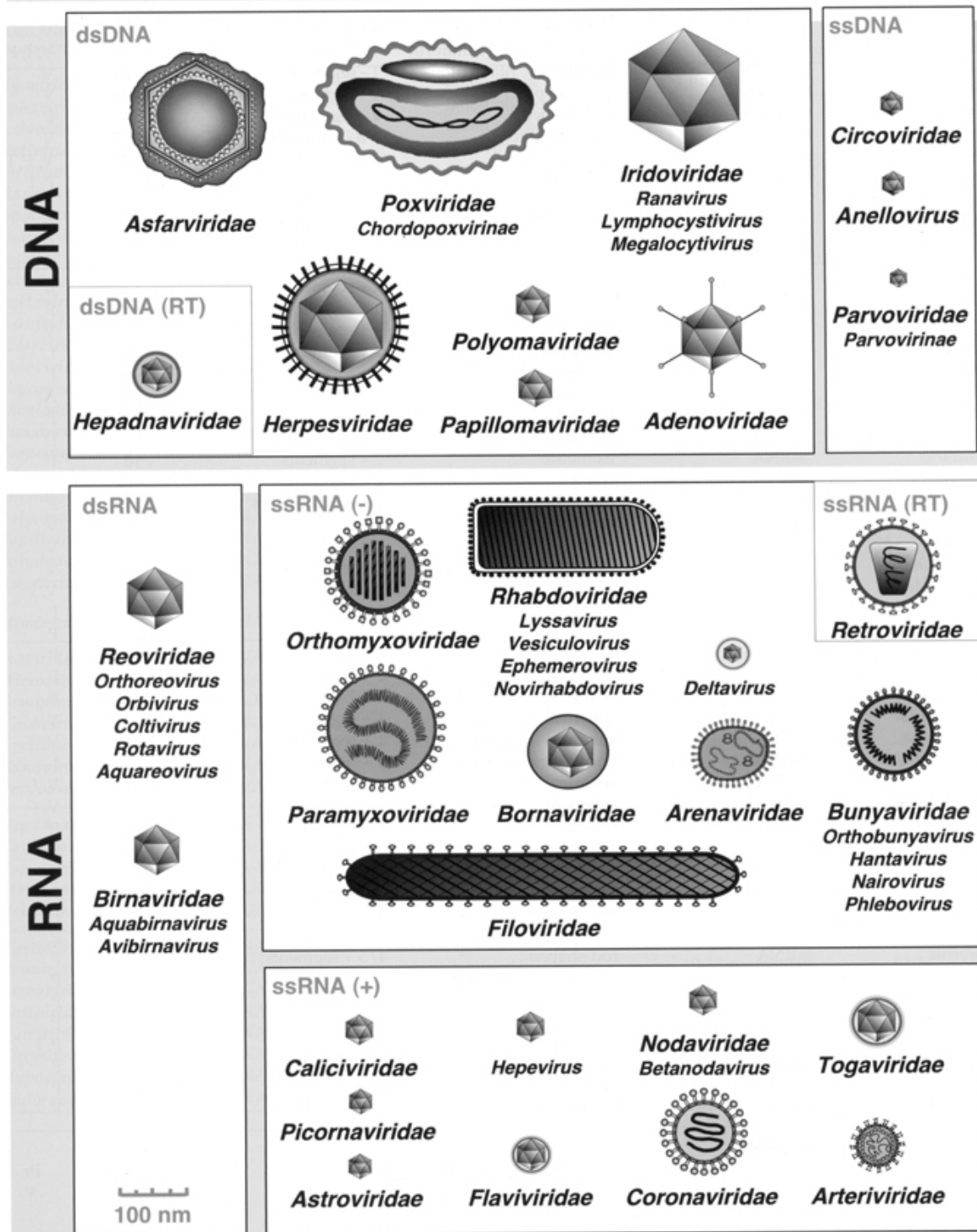
**dsRNA**



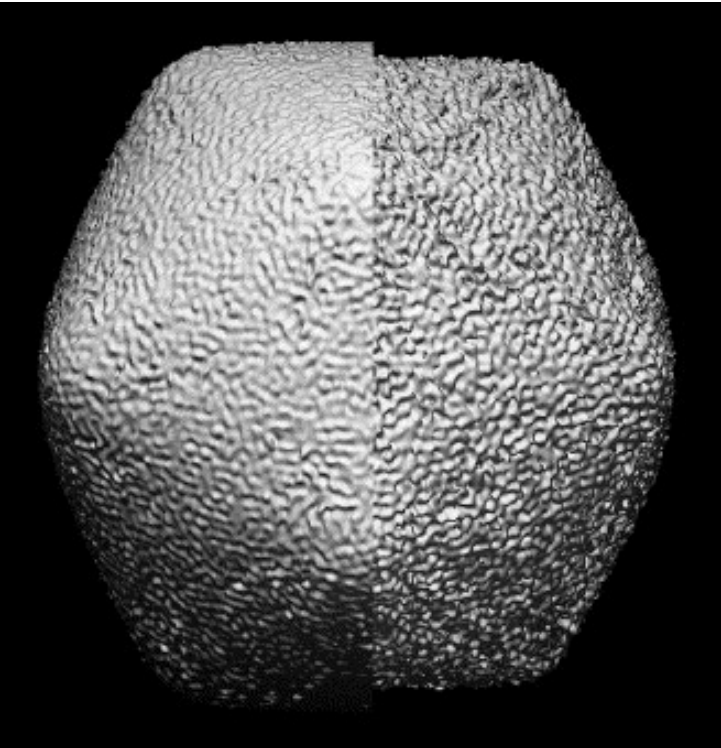
Cystoviridae

# Icosahedral Vertebrate Viruses

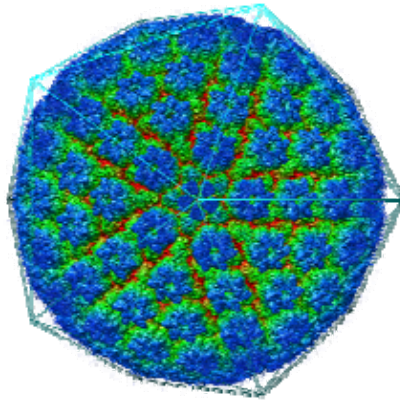
## Families and Genera of Viruses Infecting Vertebrates



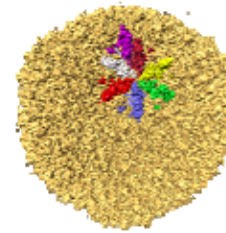
# Icosahedral Particles



Mimivirus  
5000Å



HSV  
1250Å



ε15  
700Å



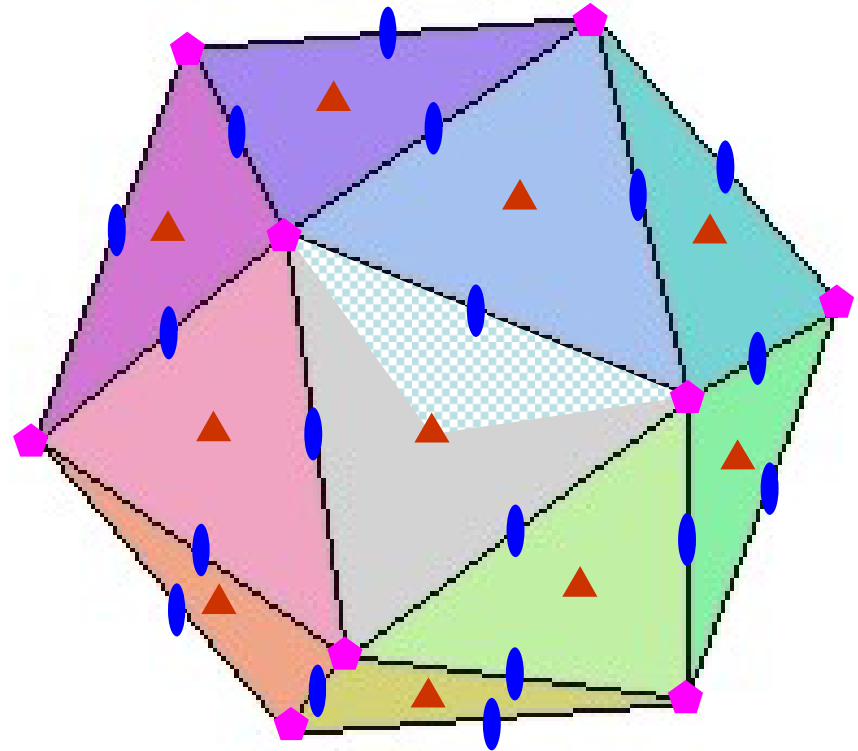
HBV  
300Å



PDC  
200Å

# Icosahedral Symmetry

- 5, 3, 2 fold axes
  - 6 five fold axes
  - 10 three fold axes
  - 15 two fold axes
- 60 fold symmetry in total
- Asymmetric unit:  $\frac{1}{3}$  of the triangular face



# Image Processing

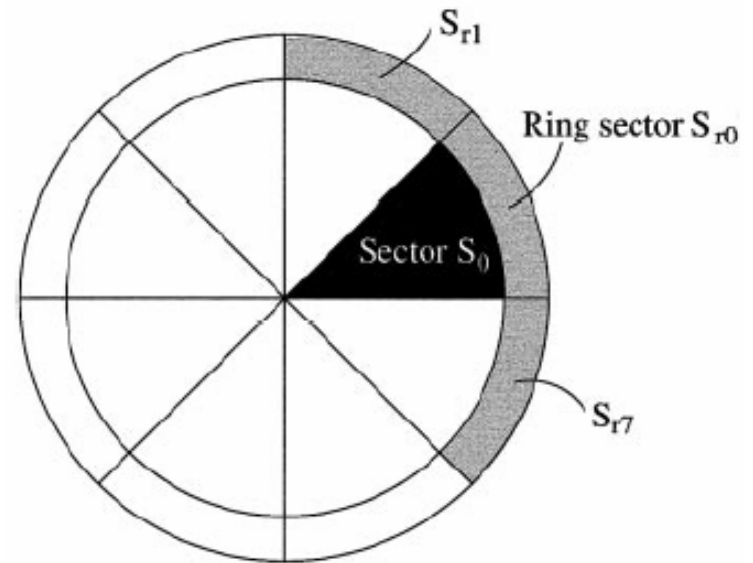
- In EMAN, icosahedral reconstruction is essentially treated the same as other symmetries
- However, one should pay special attention to the “**large**” problems
- Specialized methods exist for more efficient processing

# Image Processing

- Preprocessing
  - Particle selection
  - CTF fitting
- Build initial model
- Image refinement
  - Orientation determination
  - 3-D reconstruction

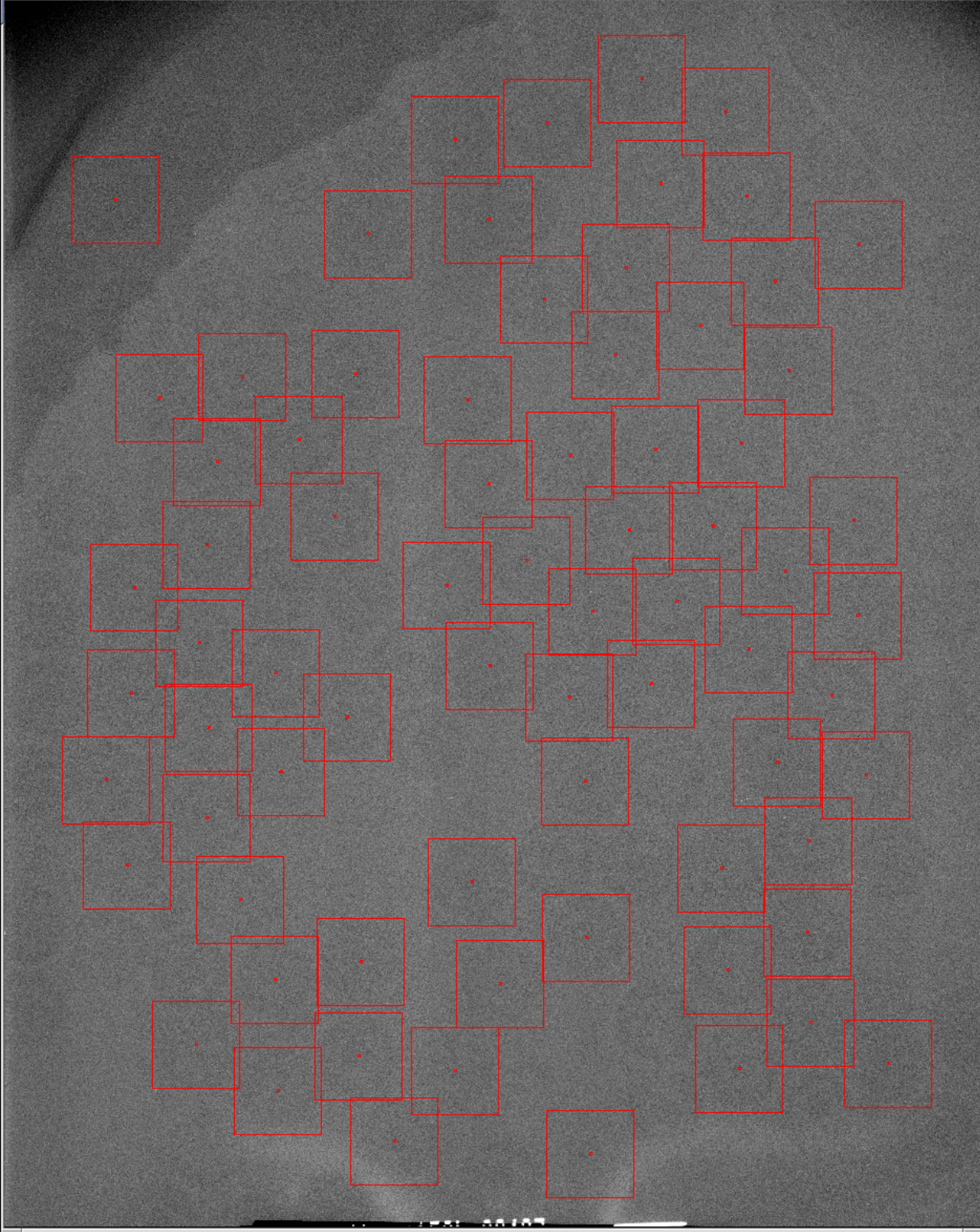
# Particle Selection

- Manual selection
  - *boxer*
- Automated selection
  - *batchboxer*
  - *ethan (for spherical particles)*



- By Kivioja and Bamford *et al.*
- “Ring Filter”
- Only **ONE** parameter (*Particle Size*)
- **Fast**, 10-15 seconds / micrograph
- Tend to over-select





# Ethan Example

Herpes  
2.1 Å/pxiel  
6662x8457 pixels  
**15 seconds in total**

```
$ time ethan.py jj0444-8bit.mrc 298 600 jj0444.box  
jj0444.img  
Opening file jj0444-8bit.mrc  
Width: 6662  
Height: 8457  
Averaging 121 pixels  
Filtering jj0444-8bit.mrc: *****  
Total number of squares is 567  
Number of peaks after sector test is 163  
Number of peaks after first distance check is 81  
Number of peaks after discarding too small ones is  
80  
Refining centers  
74 particles found from jj0444-8bit.mrc  
  
10.31s user 2.08s system 84% cpu 14.578 total
```

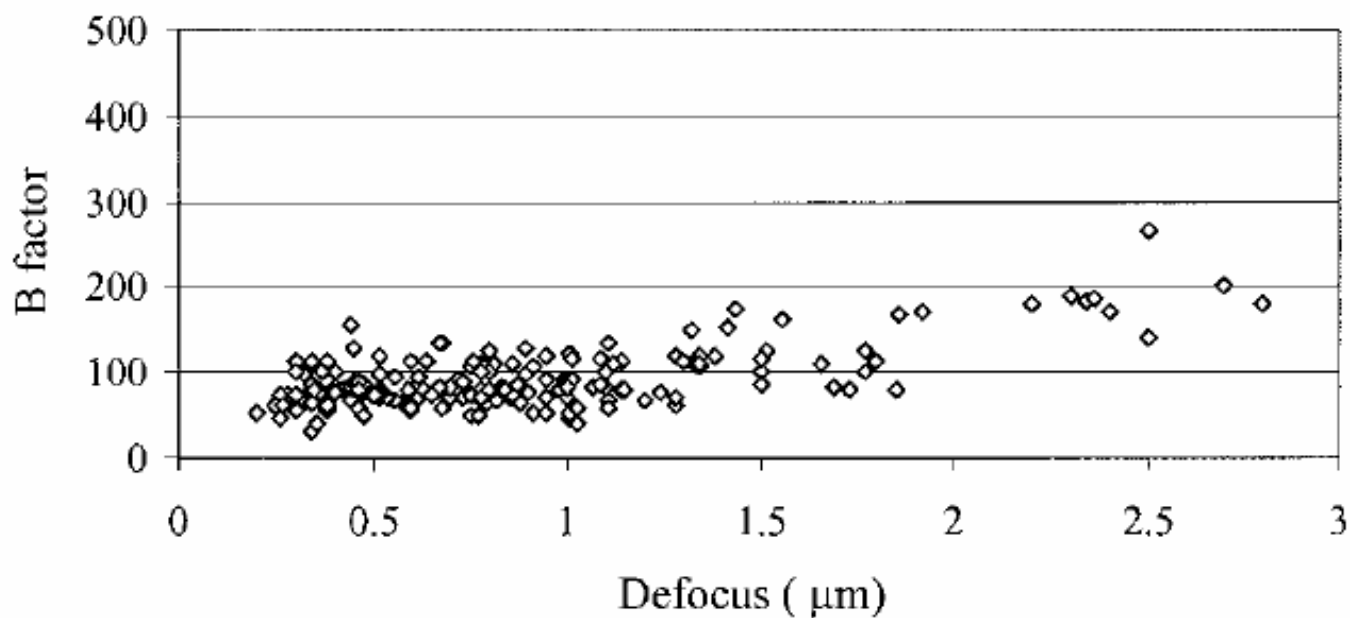
# Particle Selection

Suggested practice:

1. *Automated selection using **ethan***
2. *Manual screening using **boxer***

# CTF Fitting

To reach high resolution, small defocuses (0.5-1.5  $\mu\text{m}$ ) are often used for imaging large icosahedral particles



# CTF Fitting

- Manual fitting
  - *ctfit*
- Automated fitting
  - *fitctf*
  - *fitctf.py*

## Algorithm:

constrained nonlinear optimization using a sequential quadratic programming (SQP) method)

$$\min \left| I_{image}(s) - I_s(s)CTF^2(s)e^{-2Bs^2} - N(s) \right|^2$$

subject to:

$$I_{image}(s) > N(s)$$

$$A > 0$$

$$B \geq 0$$

$$1 \geq Q \geq 0$$

$$\Delta Z < 0 \text{ (under focus)}$$

$$I_{image}(s) = I_s(s)CTF^2(s)e^{-2Bs^2} + N(s)$$

$$ctf(s) = A(\sqrt{1-Q^2} \sin(\gamma(s)) + Q \cos(\gamma(s)))$$

$$\gamma(s) = -2\pi(C_s \lambda^3 s^4 + Z \lambda s^2 / 2)$$

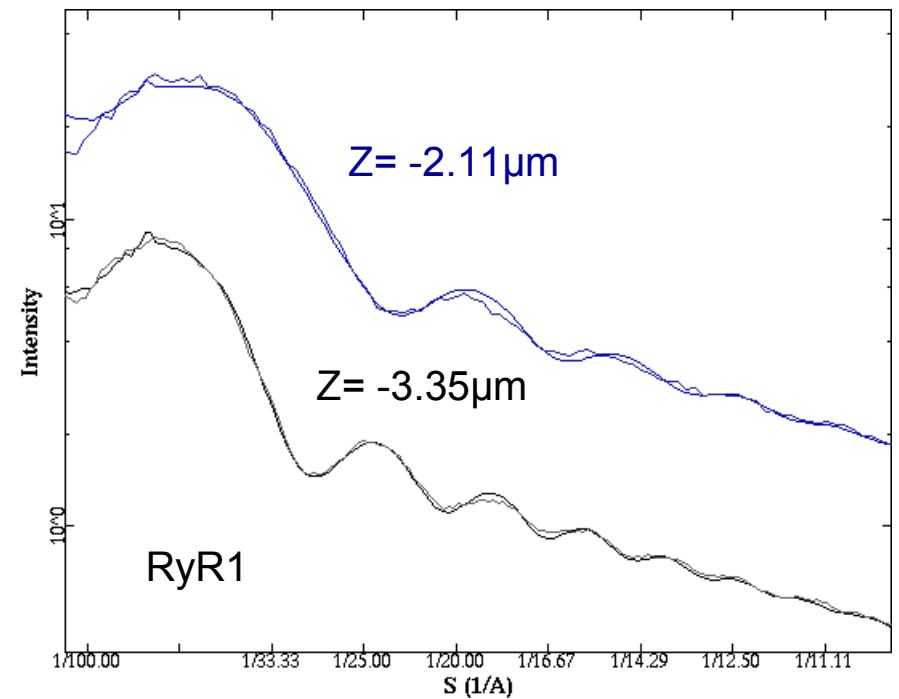
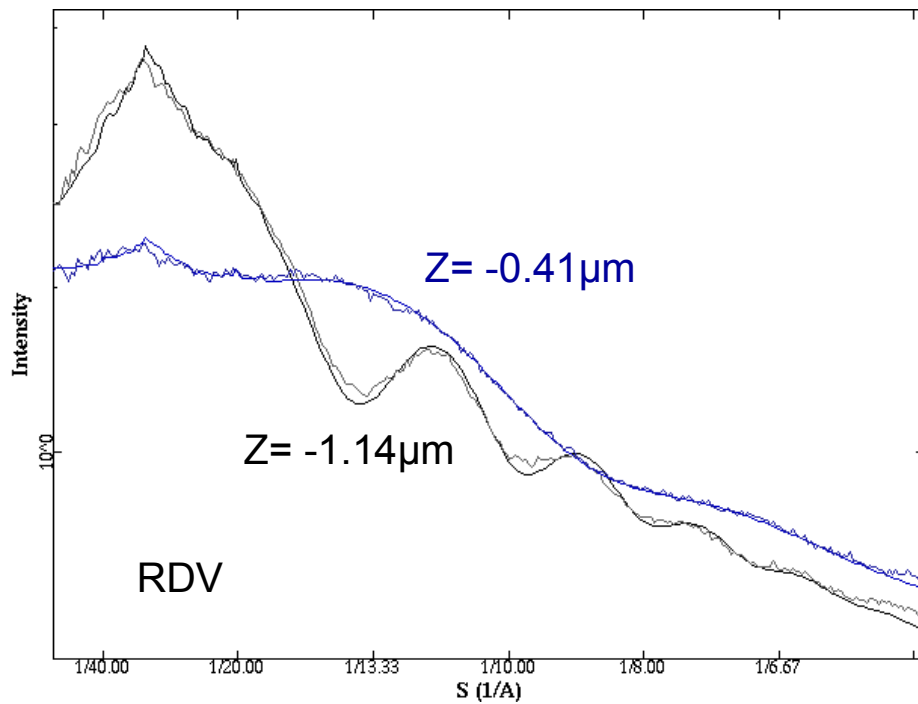
$$N(s) = n_3 e^{-n_1 \sqrt{s} - n_2 s - (m_4 s)^2 / 4}$$

8 unknown parameters:  $Z, B, A, Q, n1, n2, n3, n4$

Yang C. et al:

<http://ncmi.bcm.edu/software/fitctf>

# *fitctf.py* Examples

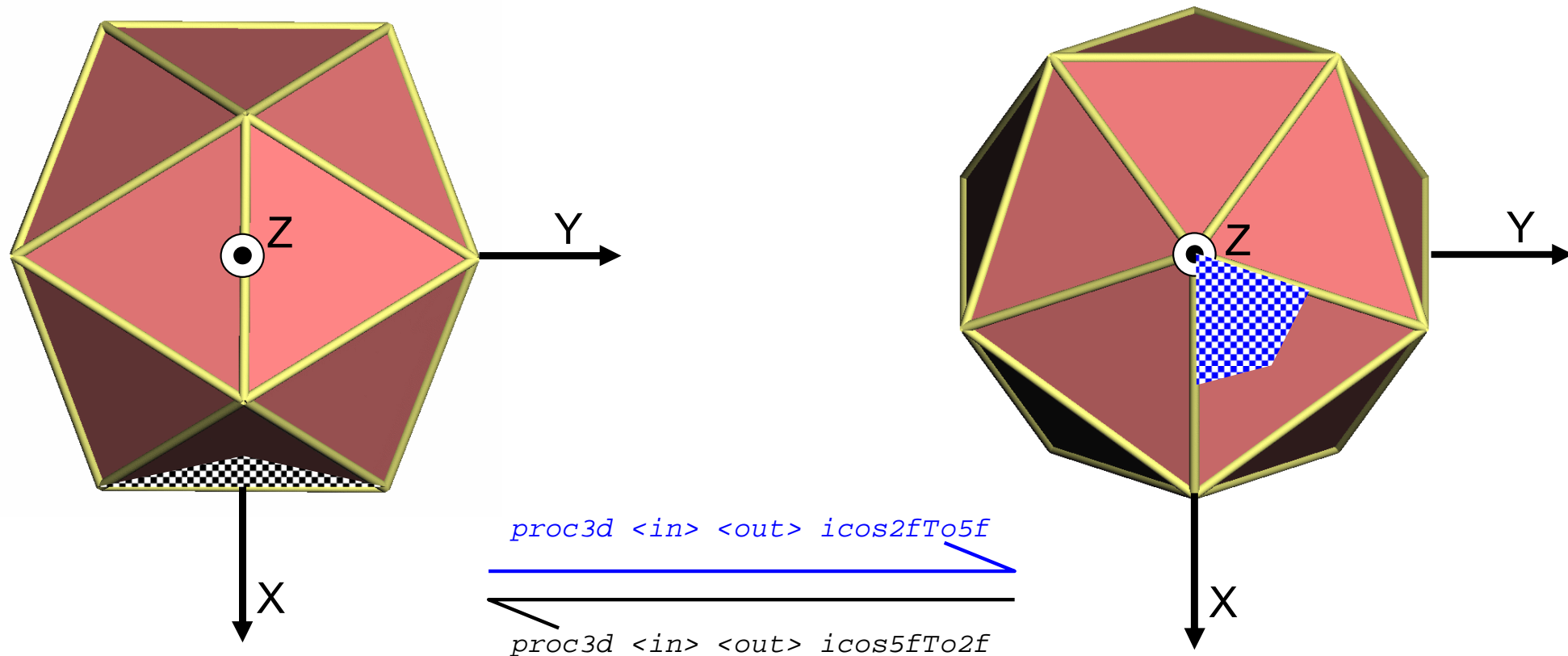


# CTF Fitting

Suggested practice:

1. *Automated fitting using `fitctf.py`*
2. *Manual screening using `ctfit`*

# Conventions: Orientation Origin



## MRC

- X, Y, Z axes along 2 fold sym axes for orientation (0,0,0)
- Euler:  $Z \rightarrow Y' \rightarrow Z''$

## EMAN

- Z along 5 fold sym axis
- Y along 2 fold sym axis
- X near 3 fold sym axes for orientation (0,0,0)
- Euler:  $Z \rightarrow X' \rightarrow Z''$

# Build Initial Model

- Classic method: self common-line
  - Small number of particle with large defocus
  - Self common-lines cluster for views near symmetry axes
- Standard EMAN method: *starticos*
  - Find best 5-, 3-, 2-fold view particles
  - Spherical particles cause problems
- New method: random model
  - Build 3-D model from particles with randomly assigned orientations
  - Relying on the wide convergence range to reach a correct structure

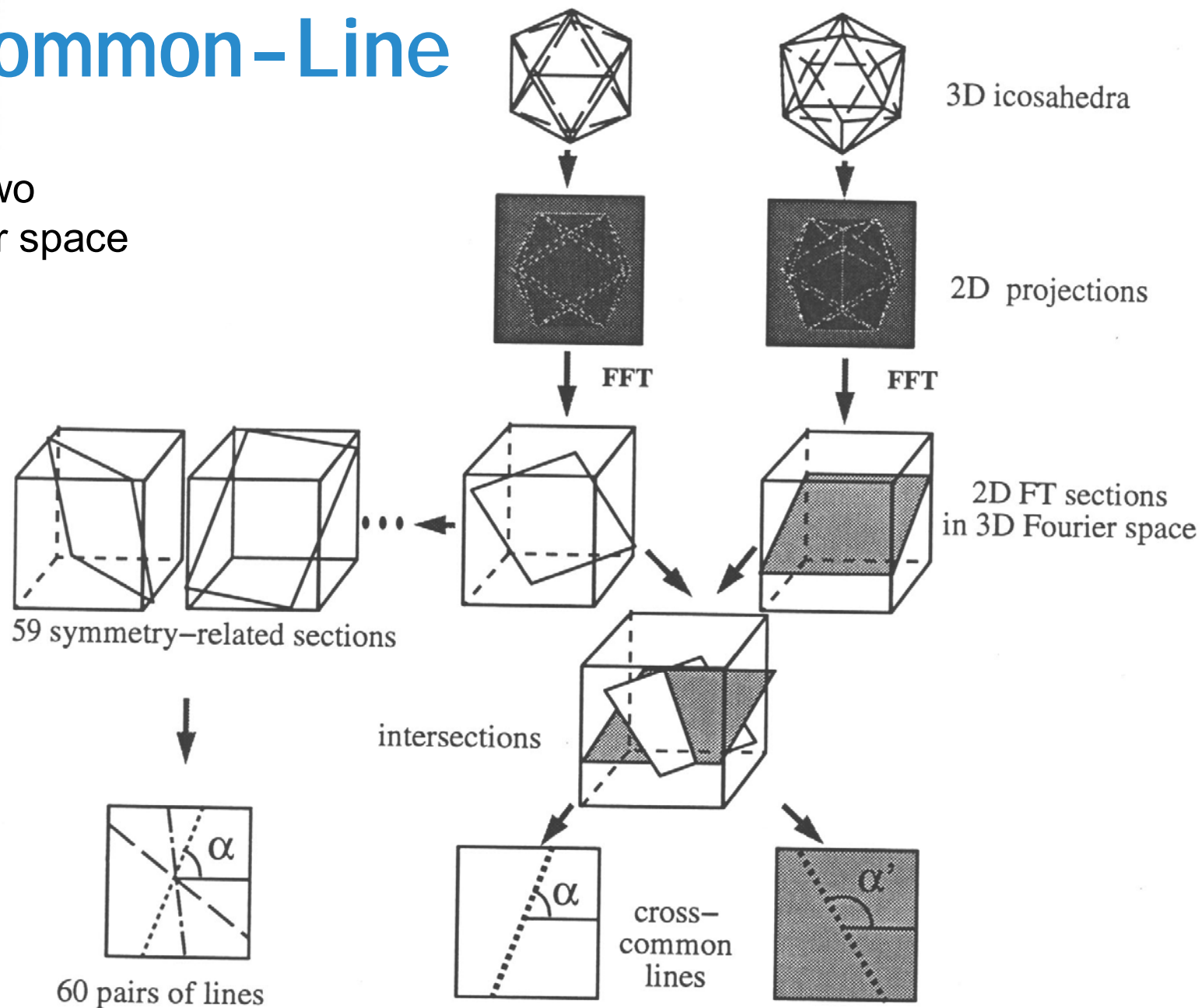


# 2-D Alignment Methods

- Common-line method:
  - Classic implementation: **self common-line** (global search to get coarse initial orientation) and **cross common-line** (local refinement) in tandem
  - EMAN implementation: Cross common-line method for both global search and local refinement (EMAN *crossCommonLineSearch.py* command)
- Projection matching method (EMAN *refine* command)

# Cross Common-Line

Intersection of two  
planes in Fourier space



# Orientation → Common Line Locations

Fourier plane normal in 3D:

$$\vec{n}_{ref} = \vec{R}_{ref} \cdot \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\vec{n}_{raw} = \vec{R}_{raw} \cdot \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

common line in 3D:  $\vec{C} = \vec{n}_{raw} \times \vec{n}_{ref}$

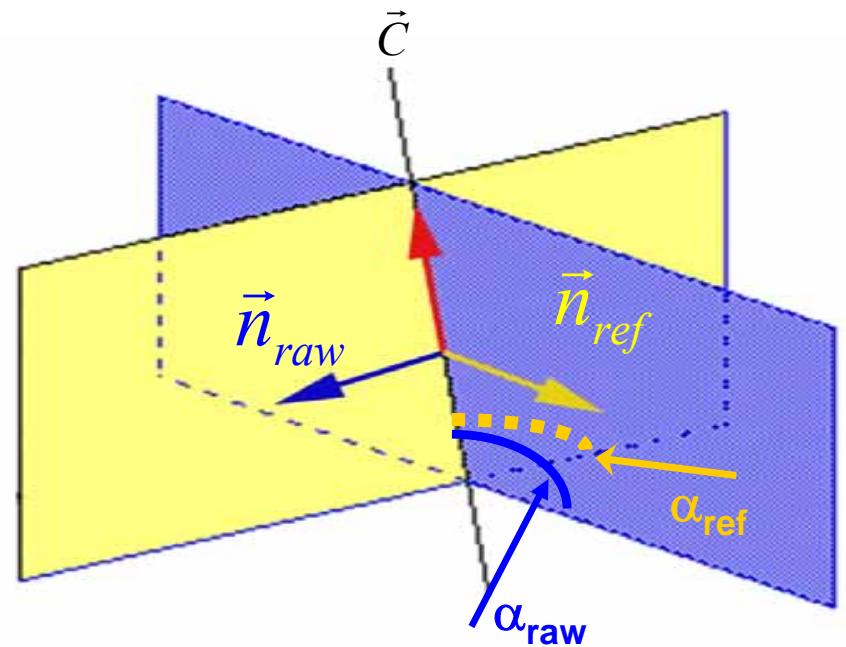
common line in 2D:

$$\vec{C}_{ref} = \vec{R}_{ref}^{-1} \cdot \vec{C}$$

$$\vec{C}_{raw} = \vec{R}_{raw}^{-1} \cdot \vec{C}$$

$$\alpha_{ref} = \cos^{-1} \left( \vec{C}_{ref} \cdot \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right)$$

$$\alpha_{raw} = \cos^{-1} \left( \vec{C}_{raw} \cdot \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right)$$



# Common Line Search: Scoring Functions

search for the **orientation** and **center** parameters that minimize the mean phase differences among all the pairs of common lines

$$R_i(\phi, \theta, \omega, x, y) = \frac{\sum_{j=1}^N \sum_{k=1}^{k_{\max}(j)} \sum_{s=s_{\min}}^{s_{\max}} f(\psi_i(s, x_i, y_i, \alpha_{i,j,k}), \psi_j(s, x_j, y_j, \alpha_{j,i,k})) \times w(s, \alpha_{i,j,k}, \alpha_{j,i,k})}{(s_{\max} - s_{\min}) \sum_{j=1}^N k_{\max}(j)}$$

$i$ : particle image

$s$ : spatial frequency

$k$ : common - lines

$j$ : reference projections

$f()$ : scoring function

$w()$ : weighting function

# Search Strategy

- Classic implementation
  1. **center** the particle by cross-correlation
  2. **self common-line** to determine initial orientation by exhaustive searching **orientation** at  $1^\circ$  step
  3. **cross common-line** to locally refine the **orientation and center** by **Gradient** or **Simplex**.

## Issues:

1. Self common-line is not very robust
2. Error propagation

# Search Strategy

- Exhaustive cross common line search
  - enumerate all centers and orientations in an asymmetric unit (5-level loops !)
  - simple but really slow
  - for 1 degree, 1 pixel step:  $3 \times 10^7$  trials/particle

# Search Strategy

- **Multi-path Simulated Annealing**
  - Multiple randomly seeded SA processes
  - Cross-communication among SA processes
  - Global search for both orientation and center
  - Accurate and fast
  - $\sim 4 \times 10^3$  trials/particle.
  - $> 10^4$  times faster than exhaustive search

# Is The Best Solution Correct?

- Absolute score (residual) cutoff
  - popular criterion
  - needs different cutoff values for images at different defocuses
  - how to pick a single number that suits all imaging conditions?

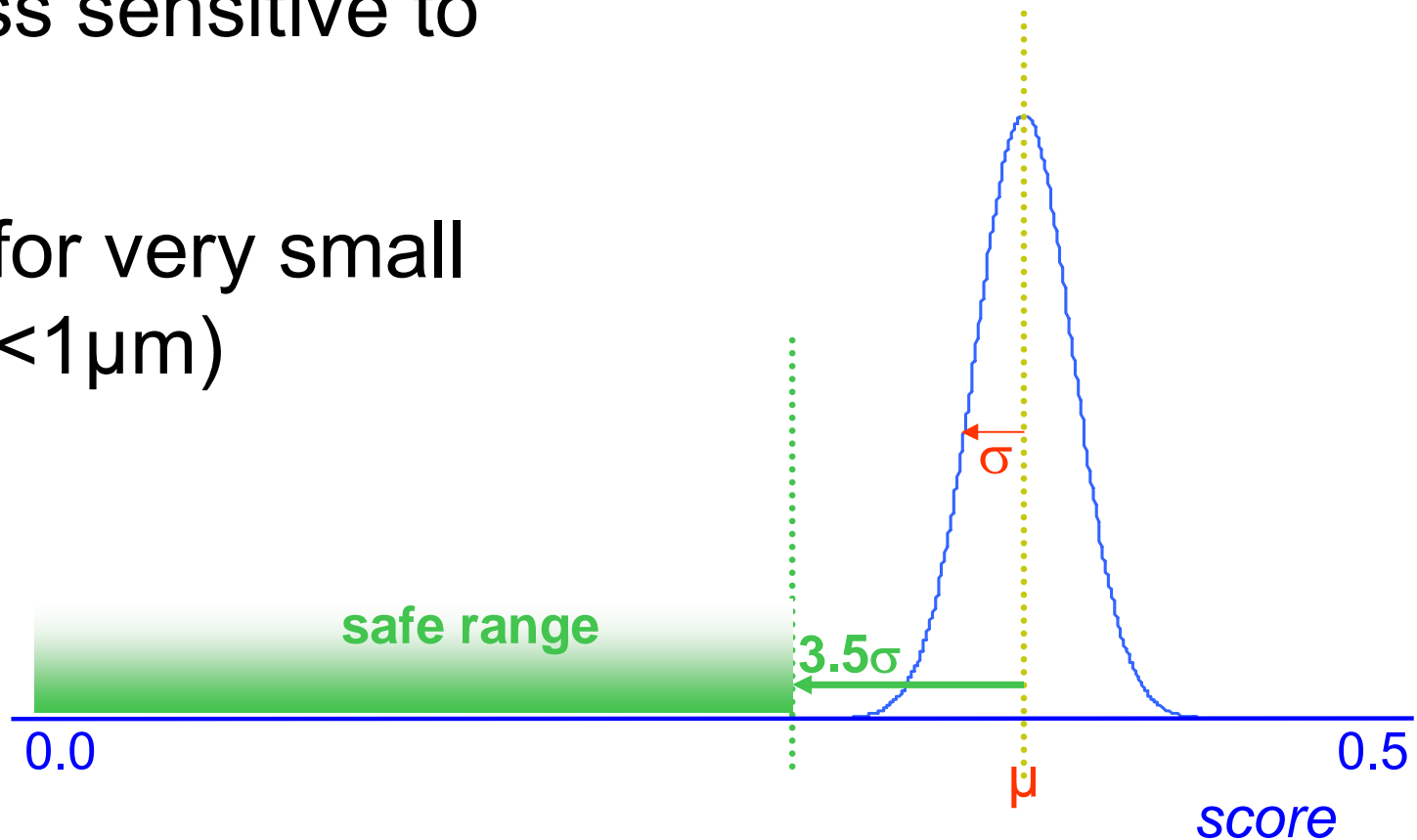


# Is The Best Solution Correct?

- Z score cutoff

$$Z = \frac{x - \mu}{\sigma}$$

- better, less sensitive to defocus
- unstable for very small defocus ( $< 1\mu\text{m}$ )



# Is The Best Solution Correct?

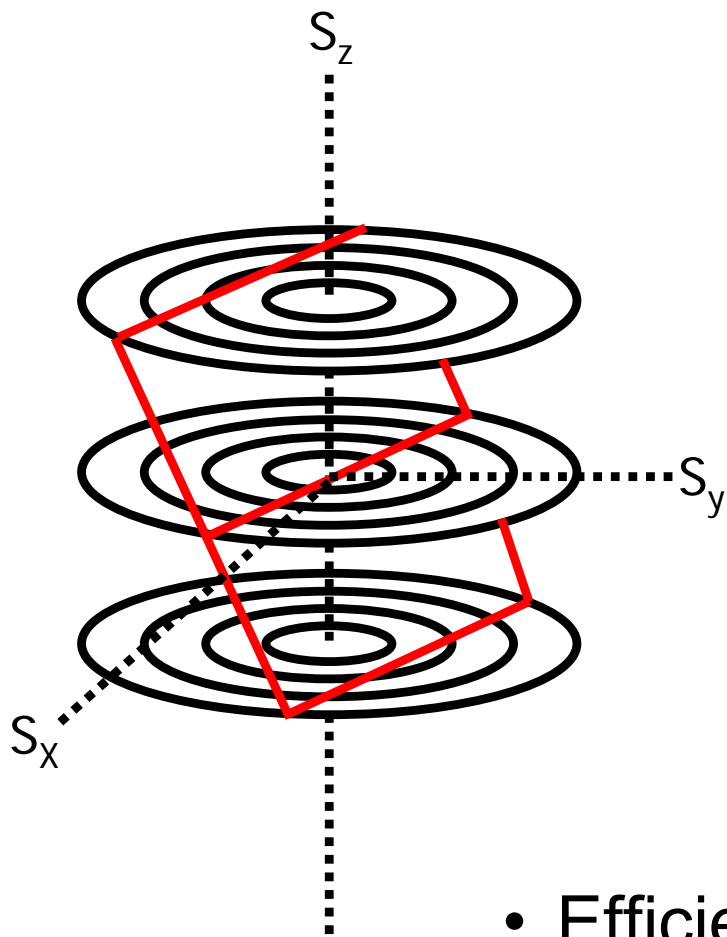
- Consistency criterion

- agreement among different methods (projection matching vs. common line)
- agreement among multiple runs of the global common line search
- more reliable than absolute value and Z-score cutoff
- bias toward “safer side”

# 3-D Reconstruction Methods

- Fourier-Bessel Synthesis:
  - Classic method for icosahedral particles
  - Very efficient in computation
- Direct Fourier Inversion
  - EMAN *make3d* command
  - Needs lot of memory

# Fourier Bessel 3-D Reconstruction



$$F(R, \Phi, Z) = \sum_{n=-\infty}^{\infty} G_n(R, Z) i^n e^{in\Phi}$$

FFT of  
2D images



$$g_n(r, Z) = \int_0^{\infty} G_n(R, Z) J_n(2\pi Rr) 2\pi R dR$$



$$\rho(r, \varphi, z) = \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} g_n(r, Z) e^{in\varphi} e^{2\pi izZ} dZ$$

3D map

- Efficient but incomplete use of data
- Anisotropic artifacts often seen

# Icosahedral Reconstruction Using EMAN

## Two approaches

- Standard way using projection matching:

*refine sym=icos <other options>*

- Cross common-line method:

*crossCommonLineSearch.py <options>*

# Special Considerations

- Load projections to memory in batches instead all at once: add “projbatches=<n>” with  $n > 1$  to *refine*
- Use half maps instead of full map to start *refine*: use “proc3d <in> <out> tophalf” to generate top half map
- Use fewer CPUs for projection and class averaging step to avoid IO timeout errors. Use “proc=16,64” instead of “proc=64” option for *refine*

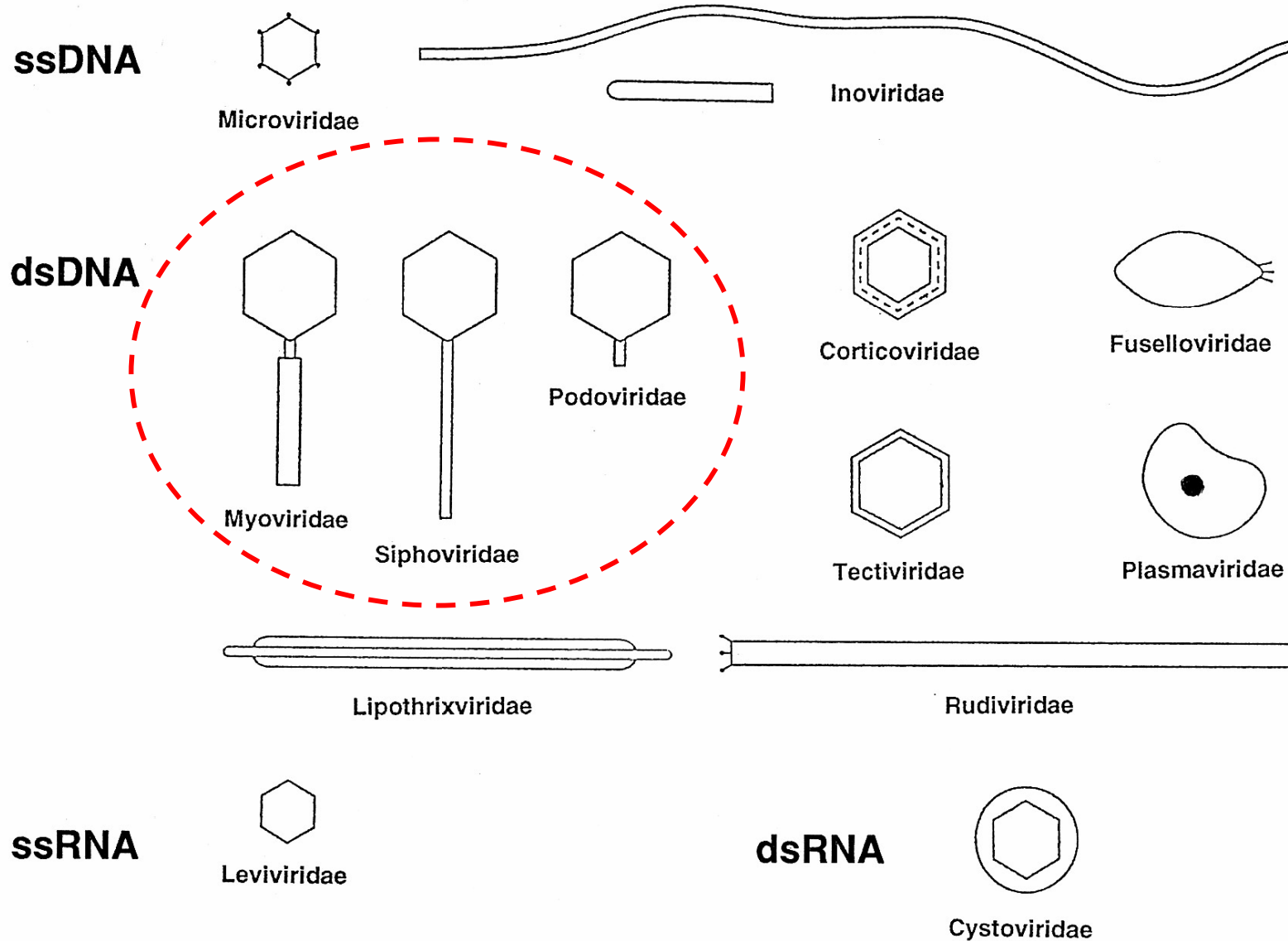
# Special Considerations

- Use parallel reconstruction instead of single CPU reconstruction. **make3d is now parallelized using mpi.**

```
mpirexec -n <# cpu> make3d ...
```

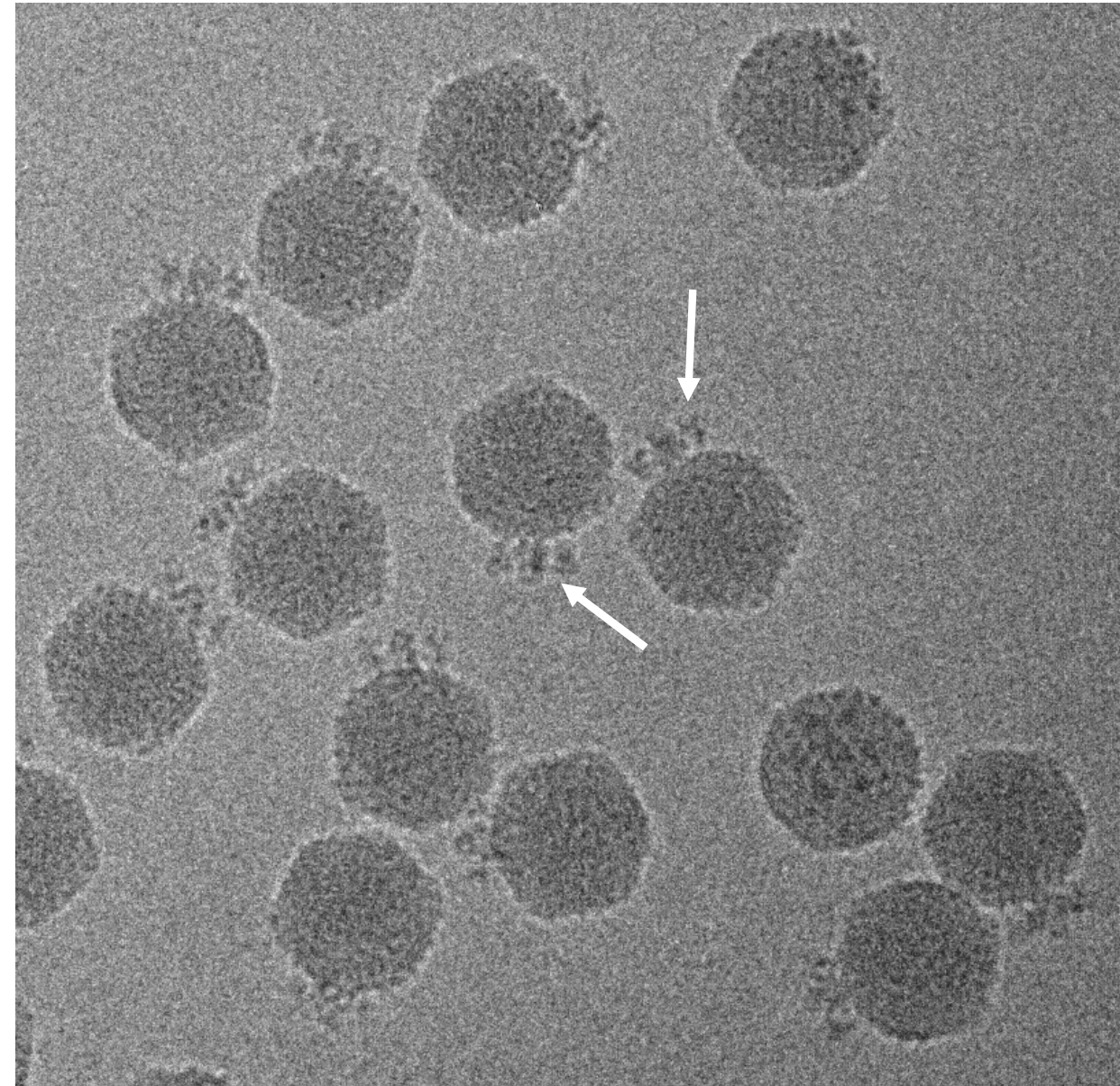
- Map size limits: **512<sup>3</sup> for 4GB, 768<sup>3</sup> for 8GB, 900<sup>3</sup> for 12GB**
- Support mixed platform processing: **one 64bit node with 12GB memory + many 32bit nodes with 4GB memory**

# Icosahedral Reconstruction → Non-Icosahedral Reconstruction





# Bacteriophage Epsilon15

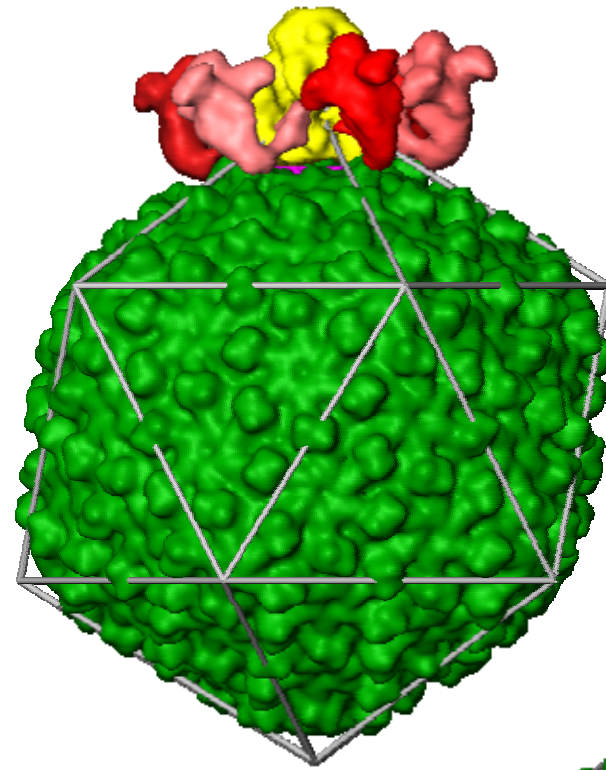
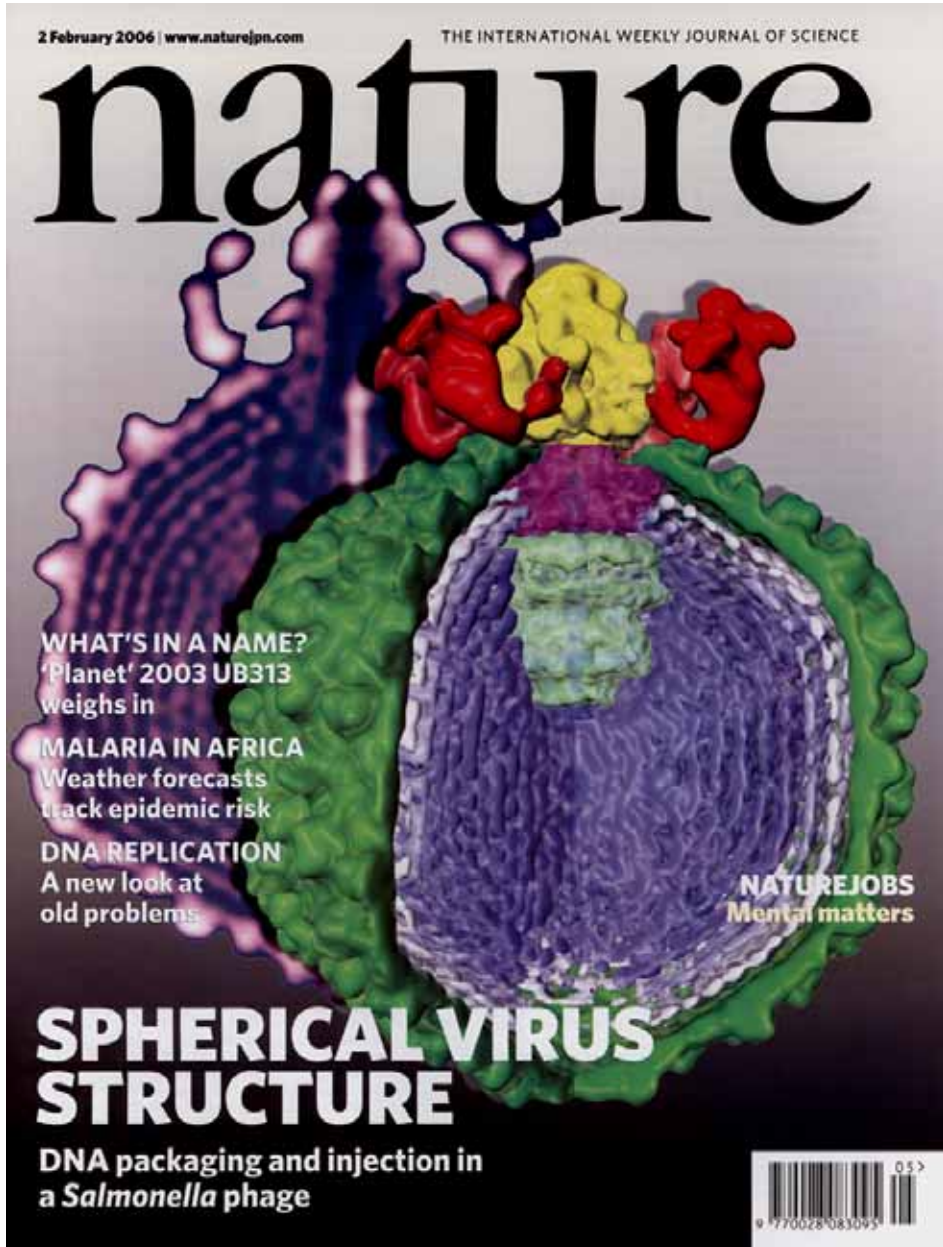


icosahedral reconstruction  
(shell only)

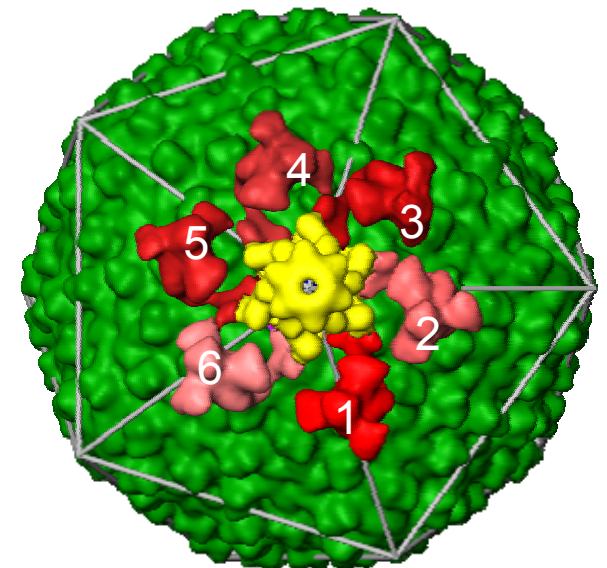


asymmetric reconstruction  
(shell, portal, tail, genome)

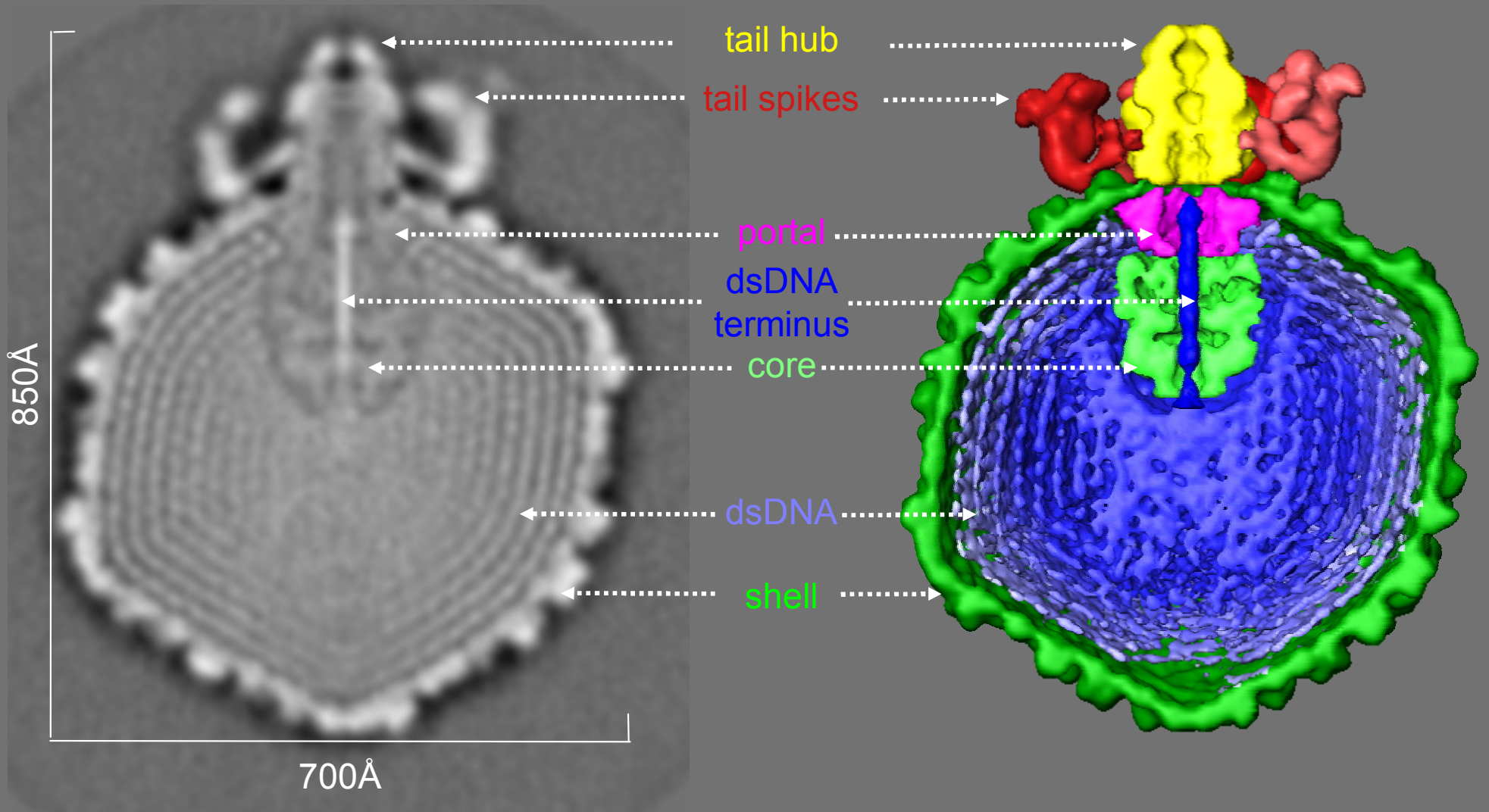
# Asymmetric Reconstruction



top view



# "Virus Anatomy"



# Icosahedral Reconstruction → Non-Icosahedral Reconstruction

*refine*

*asymrefine.py*

